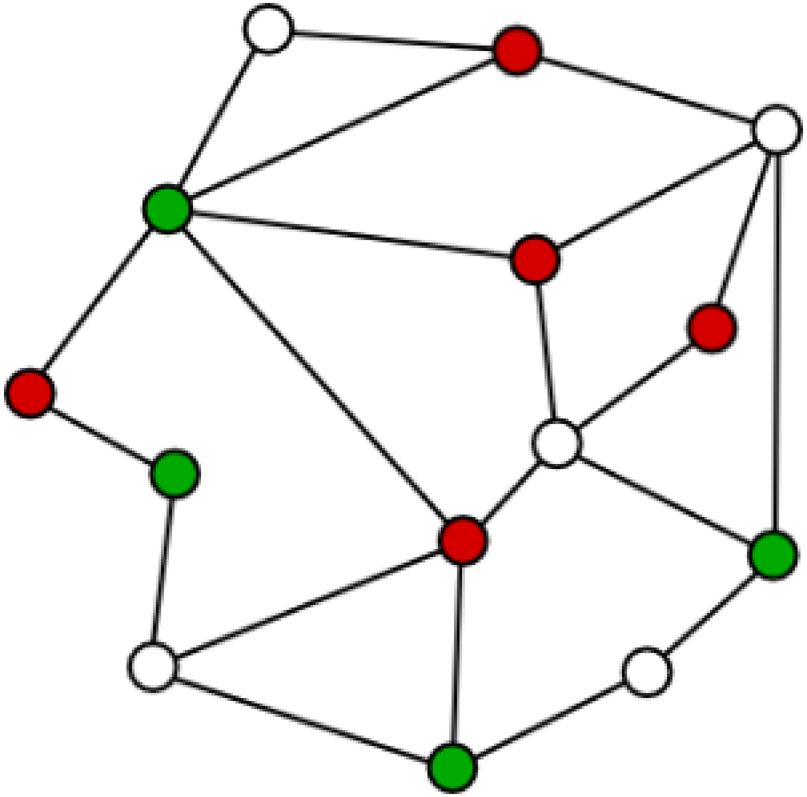
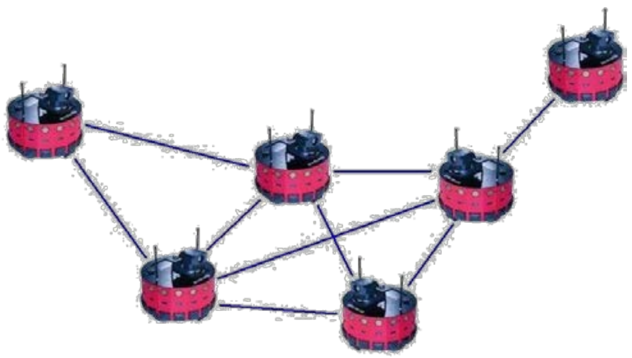


Introduction to Graph Theory

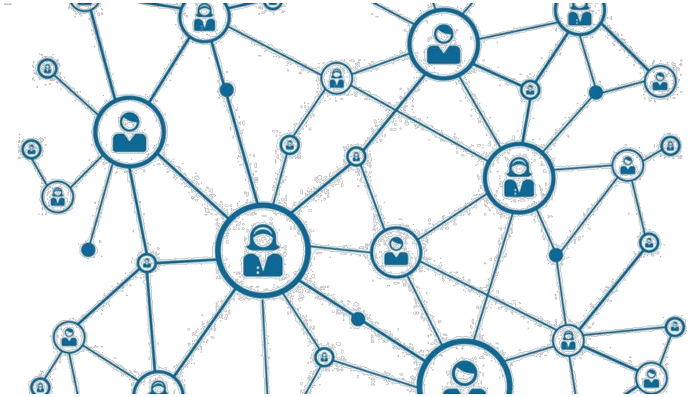
Motivation

- How to achieve **global** behaviors from **local** behaviors?



❑ **Multi-Agent Networked Systems**

- Robot/vehicle networks
- Sensor networks
- Social networks
- Power networks



❑ **Why multi-agent?**

- Agents represent the different entities (robots, vehicles, sensors, users…)

❑ **Why networked?**

- To model the limited information of each agent about the other agents, due to sensing and communication limitations

Examples

- Reynolds Boids Model
  - A collection of mobile agents collectively achieves a global task with local interactions
  - **Rules of local interactions**
    - Separation (collision avoidance)
    - Alignment (align velocity with neighbors velocity)
    - Cohesion (avoid becoming isolated from neighbors)

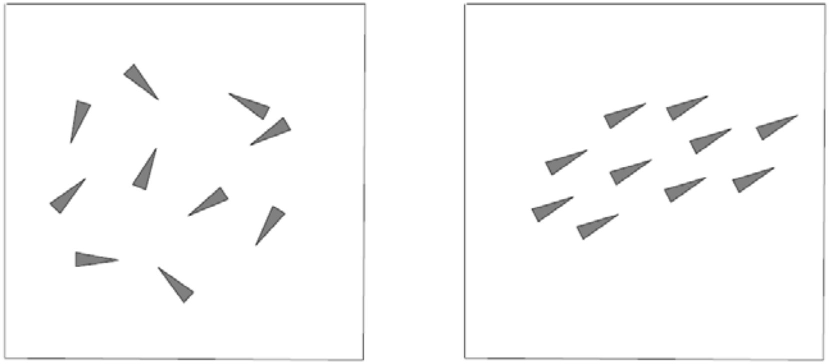


Figure 1.1: A Reynolds boids model in action. Ten agents, each with an arbitrary initial heading (given by the orientation of the triangles) and spacing, are considered (left); after a while they are aligned, moving in the same general direction at regular interagent distances (right). When this is the case, we say that *flocking* has been achieved.

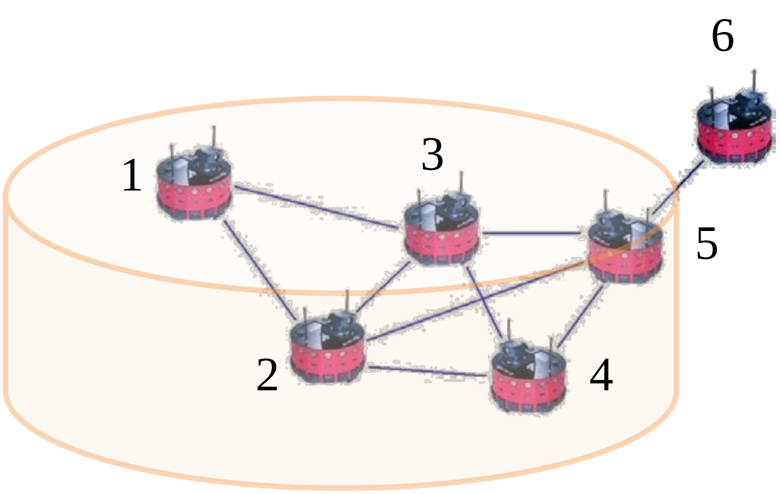
- Formation Flight
  - Distributed spacecraft systems (e.g., space interferometers, planet finders)
  - Unmanned aerial systems (e.g., for surveillance, mapping, target detection)

- More examples in Chapter 1
  - [Mesbahi and Egerstedt, "Graph Theoretic Methods in Multi-Agent Networks].

Common Fundamentals

- Common Fundamentals
  - A set of dynamic units that receive and transmit information among each other
  - A signal-exchange network (typically via wired or wireless protocols)

  - Locality in communication (e.g., limited range, limited bandwidth)
  - Locality in sensing (e.g., limited range, limited field-of-view)



- Limitations in communication and sensing do not allow each agent to (directly) share information with everyone else
  – Example: 2 does not communicate directly with 6

- **Graphs** model the available capabilities in terms of communication and sensing

- A graph G=(V,E) is a collection of vertices (or nodes) V, and edges (or links) E.

    - Agents are the **vertices (nodes)** in the graph that represents the network.
    - Pairs of agents that can exchange information are connected with **edges (links)**

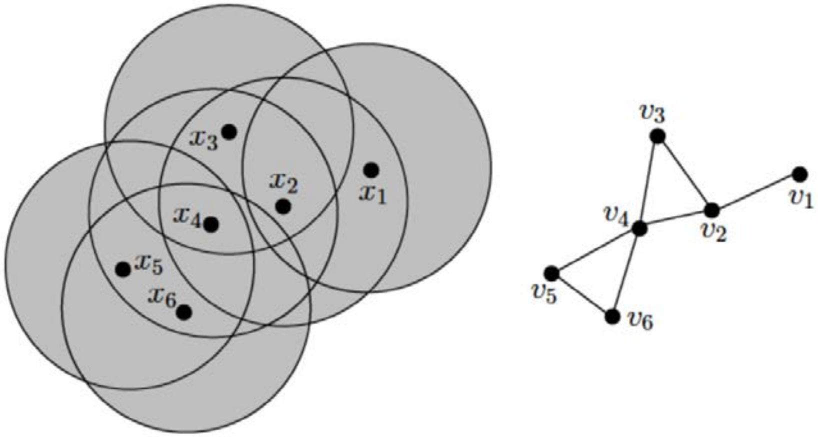
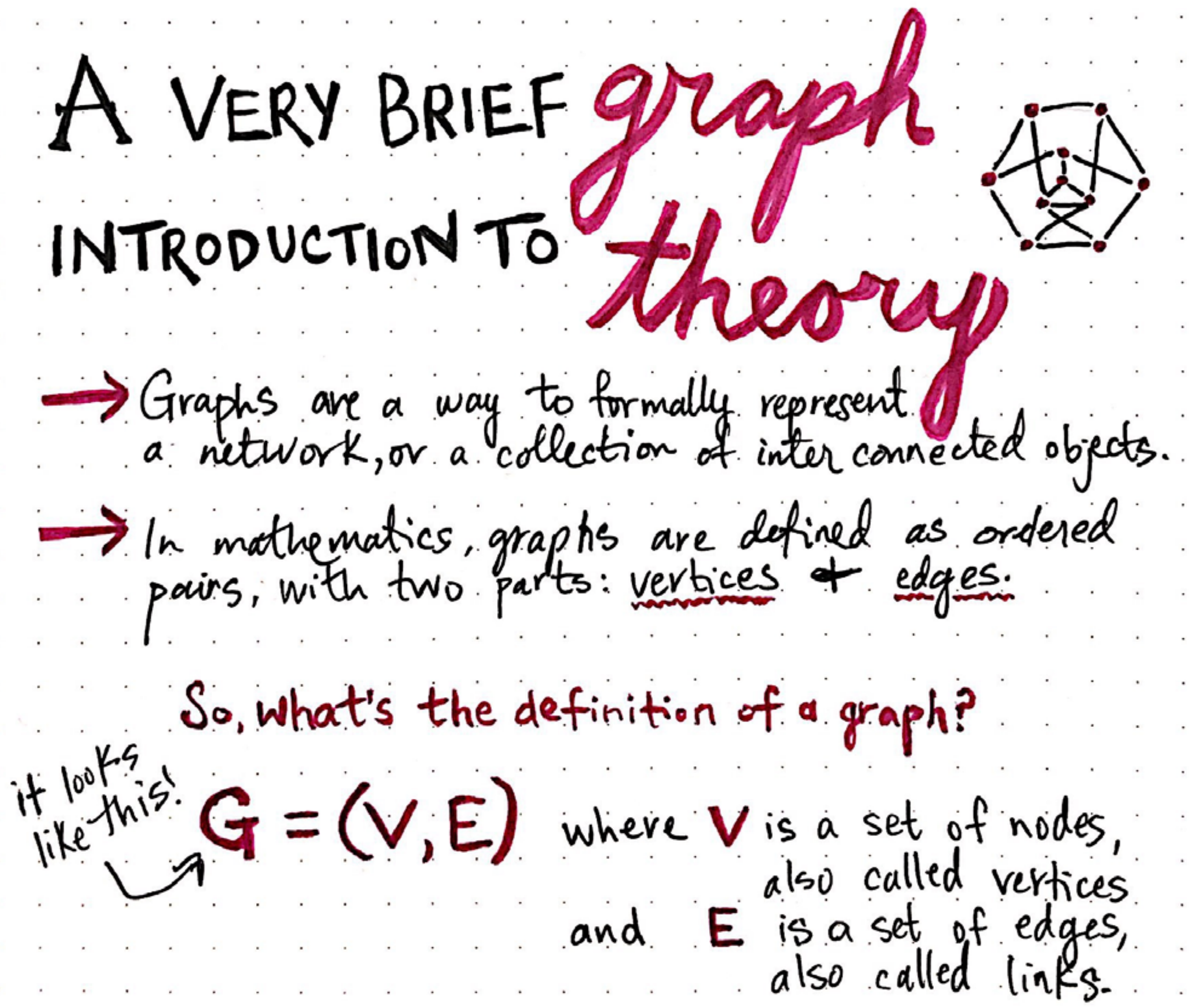


Figure 1.4: A network of agents equipped with omnidirectional range sensors can be viewed as a graph, with nodes corresponding to the agents and edges to the interactions.

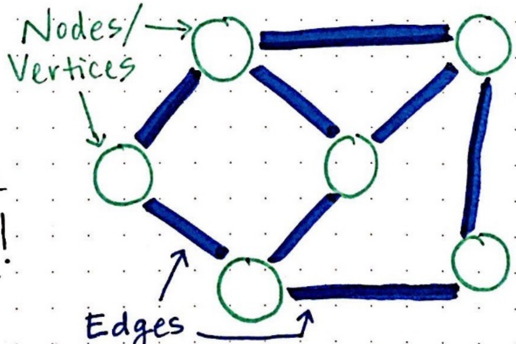
The network graph:

- Gives a high-level description of how agents (vertices) interact (are connected) through edges (links)
- Does not include exact representations of the information shared, or the communication protocol

Graphics taken from https://medium.com/basecs/a-gentle-introduction-to-graph-theory-77969829ead8

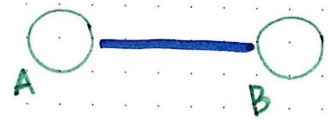


Edges can connect nodes in *any* possible way! No rules!

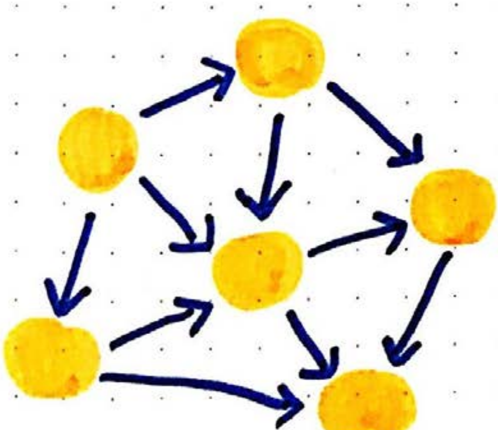


Different types of edges in graphs

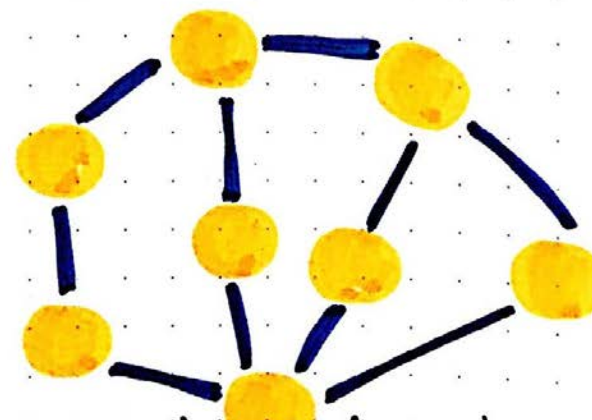
directed edge: there is only a path from A, the origin, to B, the destination



undirected edge: the path between A and B is bidirectional, meaning origin & destination are not fixed.



directed graph/digraph



undirected graph