Fréchet means and Procrustes analysis in Wasserstein space

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Motivated from the task of registering warped point processes, we consider two statistical problems at the intersection of functional and non-Euclidean data analysis: the determination of a Fréchet mean in the Wasserstein space of multivariate distributions; and the optimal registration of deformed random measures. We elucidate how these two problems arise canonically in the point process setup and how they are interlinked, each being in a sense dual to the other.

Exploiting the tangent bundle structure of the Wasserstein space, we deduce the Fréchet mean via gradient descent. We show that this is equivalent to a Procrustes analysis for the registration maps, thus only requiring successive solutions to pairwise optimal coupling problems. We additionally show how this procedure yields the optimal multicoupling, and demonstrate its stability with respect to discrete observation of the measures: we construct nonparametric estimators that are proven to be consistent for the population mean and uniformly consistent for the Procrustes registration maps.

Our techniques are tailored to the Wasserstein geometry, because Hessian-type methods from non-Euclidean statistics do not apply to the Wasserstein space.

This is joint work with Victor M. Panaretos (École polytechnique fédérale de Lausanne).